RELIABILITY ENGINEERING

18 7301 - ME (18E)

Reliability:

⇒ Probability of the product (or) item (or) Component to perform Specified Operating Condition Under the time period.

Basic elements:

* Probability * Performance level
* Duration * Operating Condition.

1) Probability: ⇒ $\frac{\text{Num of time Event Expected}}{\text{Total Num of Event (trails) Undertaken}}$

Assumed

Reliability max Value - 1

min Value - 0

∴ The Probability Value is one → Reliability perfect

Value zero the system Never Work

So the system performance Judged for,

→ duration

→ Operating Condition

Example:

* Reliability of cylinder Relief Valve of Boiler is zero 100 safety valve tested under Identical conditions

Result:

None of them will work

* Reliability value one

Result:

All of them work satisfactorily

Adequate performance:

(Expectation of a device Resulted in Successful Rate)

A 5 amp fuse to perform if the fuse blows off when current passing is exceeds of 5 amp

Duration:

It is the time period for which the product is satisfactory

Operating condition:

Evironmental factors, Temp, vibration, pressure, shock etc..

Example:

AC designed to perform Hot and dry Climate.
But will not perform wet Climatic Condition

Another definition

Reliability:

Ability of an item to perform a required function under stated condition for stated period of time.

REPAIRABLE ITEMS:

Repairable item is one which can be put back in service Economically.

Ex: Engine, Aircrafts, Camera, MIC,

Non - Reparable Item:

which fails Compoletly.

* It is an Individual part

Ex:

Bulb, transistor

* Cannot be Reuse, the life circle fully Compcted.

Tasic of Reliability:

Three tasic

↳ To study Existins system in detail

↳ To determine Best way of Increasing Reliability

↳ To maxmise system Reliability with in condition.

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Reliability Related Terms:

→ failure → Maintainability

→ Availability → Reliability

1) Failure:

failure is an termination of the ability of an item to perform a Required function.

→ failure Rate (or) Hazard Rate:

$$Z = \frac{\text{Num' of failures during a particular unit Interval}}{\text{Avg population}}.$$

Mean Time B/W failure (MTBF):

* The Mean time b/w two Successive Component failures.

Normally taken to be the mean b/w nth & (n+1)th failure in a system.

n → Large.

MTBF ⇒ $\dfrac{\text{Cumulative observed time}}{\text{Num. of failure under stated condition}}$.

MTBF is an Repairable items.

Mean time to failure (MTTF).

MTTF = $\dfrac{\text{The Cumulative time for sample}}{\text{total Num. of failure in the sample}}$.

MTTF → Non Repairable items.

Maintenability.

The probability of unit (or) system will be restored in specific Condition with in a specific period.

When the Maintenance action taken with prescribed procedure and Resources.

Maintenance

Corrective

Preventive.

Two terms Relatively to Reliability

→ Maintainability

→ Availability

Availability:

→ Operational Readiness of the system.

Ability of an Item to perform its Required function at a Over a stated period of time.

* The System has High Maintenability that is Easy to Repair and Maintenance

* Availability will be more the system break down less

* Maintainability depends upon the design & installtion of the system.

Types of Availability:

→ Inherent Availability (Ai)
→ Achieved Availability (Aa)
→ Operational Availability (A o)

Availability:

the ratio of equipments up time to the equip (up time + down time) for specific period.

uptime → m/c (or) system working desired condition.

Down time → Unacceptable working condition.

System Effectiveness:

system Reliability → Availability × main. × Reliability × Capability.

⇒ AMRC

Markov Analysis:

→ It is an provides a method of Analyzing the Reliability and Availability of sub - systems Representing Components with Strong Interdependencies

* The markov analysis also analyzed independly of the FTA.

* The markov models Representing the Continuous (or) discrete translations.

Features:

* Graphically Constructed transition diagram

* Global Parameter facility for Repetitive data.

* Calculation of a wide range of probabilities and frequencies.

Two Components Connected in Series System.

$R_s(t) = P_1(t)$

Two Components parallel System.

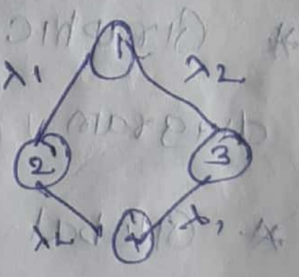
$R_p(t) = P_1(t) + P_2(t) + P_3(t)$

$\therefore \; P_1(t) + P_2(t) + P_3(t) + P_4(t) = 1 \longrightarrow (1)$

$i = 1, 2, 3, 4 \ldots$

Assume,

Individual Component have Constant failure rate λ_i.



As per rate diagram

$P_1(t+\Delta t) = P_1(t) - \lambda_1 \Delta t\, P_1(t) - \lambda_2 \Delta t\, P_1(t) \rightarrow (2)$

$P_2(t+\Delta t) = P_2(t) + \lambda_1 \Delta t\, P_1(t) - \lambda_2 \Delta t\, P_2(t) \rightarrow (3)$

$P_3(t+\Delta t) = P_3(t) + \lambda_2 \Delta t\, P_1(t) - \lambda_1 \Delta t\, P_3(t) \rightarrow (4)$

Life cycle Cost Analysis:

It is a technique for Calculating the whole life cost of a Plant. of system from Inception to disposal.

It Can be used any Industry. the system involved Could be a building, a ship a weapons system (or) power plant

Reliability Allocation:

The first step in the design process is to translate the Overall system Reliability Requirements into Reliability Requirements for Each of the sub system. this process is known as Reliability Allocation.

www.Reliability analytics.com

www.weibull.com

for General Eqn:

$$f(R_1, R_2, R_3) \geq R^*$$

R^* → system Reliability

f → Functional Relationship b/w subsystem and system Reliability

$\leq$ - less than

$\geq$ - Greater than

Simple Series system Represent probability of Survival for t- hours

$$R_1(t) \cdot R_2(t) \cdots R_n(t) \geq R^*(t)$$

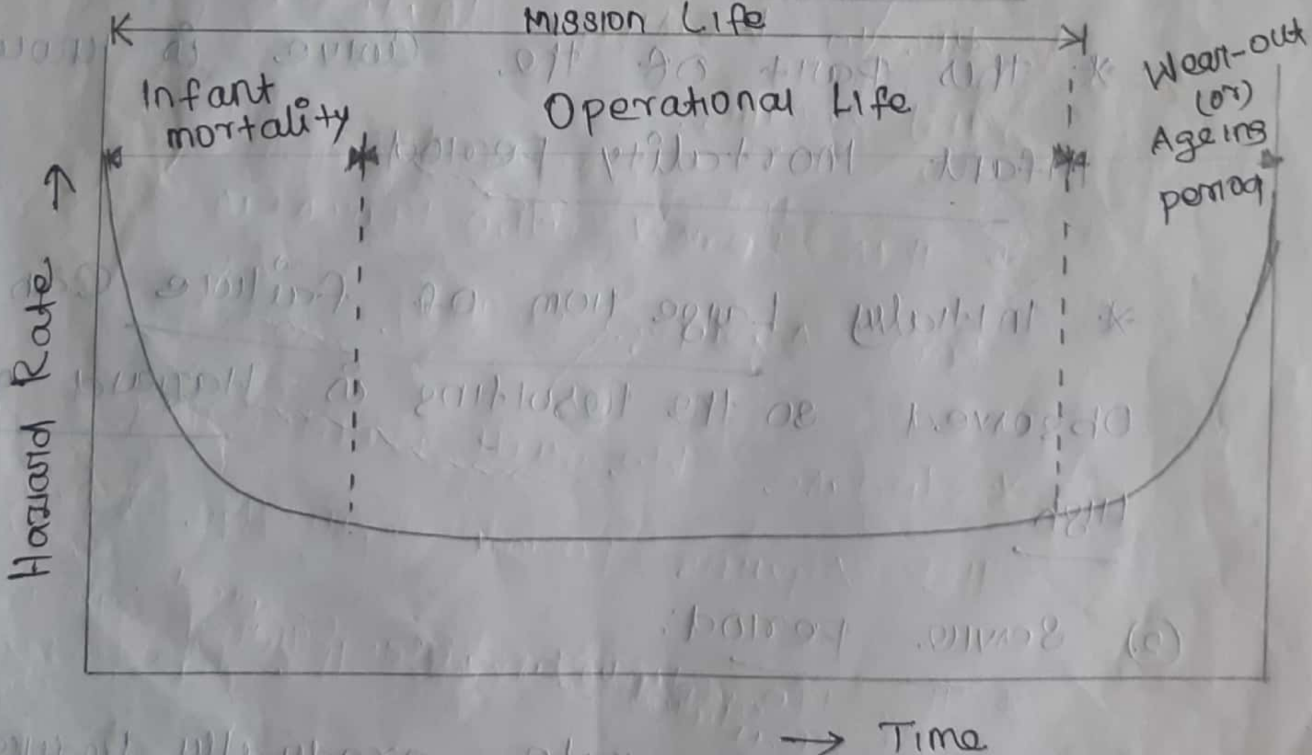
(2)

BathTub Curve:

* Bath tub Curve used to seen that life time of Entire product (or) system.

* We Conduct life test on any type of product (or) system.

* Due to its special shape, the Curve is popularly known as Bath tub Curve in the Reliability literature.



Explanation:

* The Curve has three distinct parts

→ Infant (or) early failure

→ Operational Life (or) Service period.

→ Wear-out period (or) Ageing period.

① Infant Mortality Period:

⇒ The Number of failure observed during initial period.

⇒ Due to Bad design, poor Quality, wrong manufacturing process, inadequate material selection.

* this part of the curve is known as Infant Mortality Period

* Initialy Large Num of failure can be observed so the Resulting is Hazard rate High

② Service period:

* The Hazard rate gradually Reduced and stabilized to maintain the constant value

* The failures are random and are Evenly distributed.

* The failure in second zone are termed as Service failure.

③ Wear - out failures:

* The Hazard rate again start to increased.

* The more failures are also observed, due to wearing, material deterioration and ageing.

* This period also known as wear - out (or) Ageing period.

* Incase we will maintenance carriedout the Ageing period can be delayed.

* The life time of the product should be completed. after using system the failures can be occured.

SYSTEM RELIABILITY

A system Consists of more than one Element (or) unit Connected in parallel, in series (or) in mixed Configuration.

* The Reliability of the Entire system is called as system Reliability

Type of Configuration:

Configuration System Classified into 3 types

- Series Configuration
- Parallel Configuration
- Mixed Configuration.

① Series Configuration:

* The Various Components are Connected in Series.

* All the Components are Inter-Connected

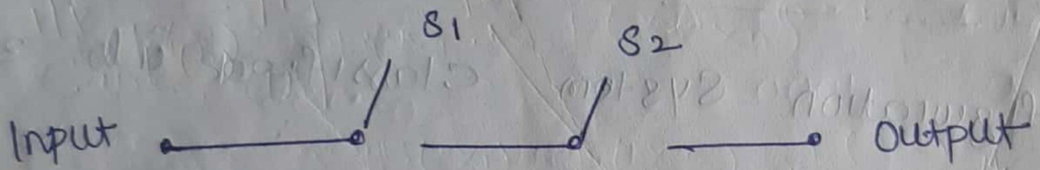
* The Entire System will fail Even If one of its Component fails

Series Configuration - deals with

- series with two units
- series with n units

a) Series system with two units:

* Consider a system in which two switches S_1 and S_2 are connected in series



Let,

$X_1 \Rightarrow$ Successful Operation of switch (S_1)

$X_2 \Rightarrow$ Successful Operation of switch (S_2)

$P(X_1) \Rightarrow$ probability of success of switch (S_1)

$P(X_2) \Rightarrow$ probability of success of switch (S_2)

$P(S) \Rightarrow$ probability of success of the system.

$\therefore P(S) = P(x_1 \text{ and } x_2)$

Note:

The two Case in Series Configuration.

Case : 1 ⇒ Units depend on one another

case : 2 ⇒ Units Independent of Each other.

Case : ①

Successful operation of unit is dependent on successful operation of the previous unit

$\therefore P(S) = P(x_1 \text{ and } x_2$

$P(S) = \boxed{\Rightarrow P(x_1) \cdot P(x_2/x_1)}$

AND

Case : 2

Successful operation of Each unit is Independent of the remaining unit

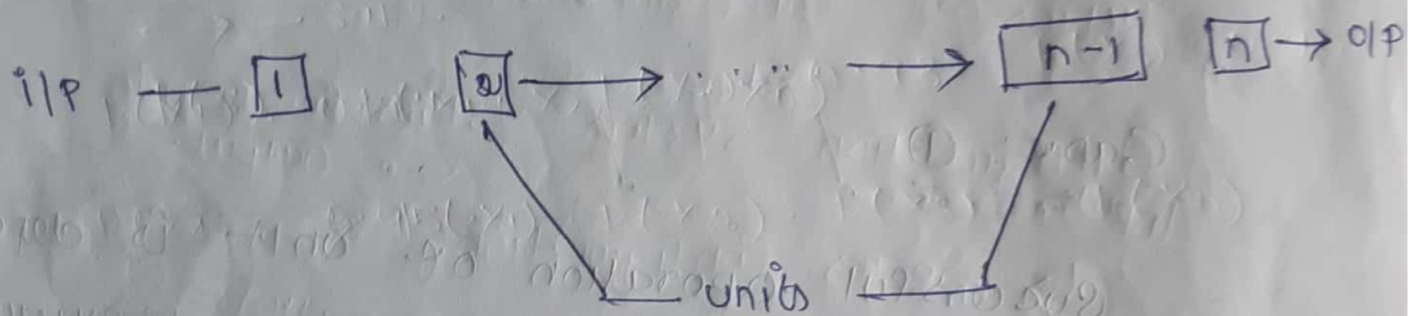
$x_1, x_2 \rightarrow$ Independent Events.

$P(S) = P(x_1 \text{ and } x_2)$

$$\boxed{P(S) = P(x_1) \cdot P(x_2)}$$

2) Series system with 'n' units:

The system with 'n' units connected in series



$P(S) = P(x_1 \text{ and } x_2 \text{ and } x_3 \ldots x_n)$

Case: 1 ⇒ units dependent on one another

Case: 2 ⇒ Units Independent of each other.

Case: 1

→ Same definition →

$x_1, x_2, x_3 \ldots x_n$ are dependent on each other.

$P(S) = P(x_1, x_2 \cdots x_n)$

$P(S) = P(x_1) \cdot P(x_2/x_1) \cdot P(x_3/x_2 \cdot x_1) \cdots$

$P(x_n/ x_1, x_2 \cdots x_{n-1})$

Case: 2

→ same definition ←

$P(S) = P(x_1 \cdot x_2 \cdots x_n)$

$\Rightarrow P(x_1) \cdot P(x_2) \cdot P(x_3) \cdots P(x_n)$

Problem: 1

Four units are Connected in Series with independent performance. determine system reliability.

Unit.	1	2	3	4
probability of Success	0.97	0.93	0.90	0.95

Given data:

$P(x_1) = 0.97$ $P(x_2) = 0.93$ $P(x_3) = 0.90$

$P(x_4) = 0.95$

To find:

$P(S) = ?$

Solution:

The system are connected series with Independent performance.

$$\therefore P(S) = P(x_1) \cdot P(x_2) \cdot P(x_3) \cdot P(x_4)$$

$$\Rightarrow 0.97 \times 0.93 \times 0.90 \times 0.95$$

$$\boxed{P(S) = 0.771}$$

problem: 2

A system has 100 units in series, Each having a reliability of 0.98. what is the Reliability of the system.

Given:

$n = 100$

$P = 0.98$

Soln:

The system connected series and each unit same Reliability

$R_{sys} = p^n$

$= (0.98)^{100}$

$= 0.1326$

Paraller Configuration:

The units are Connected in parallel

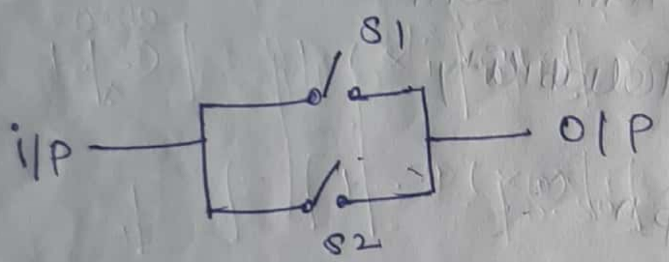
→ Remaining Same ←

Paraller system with two Units

paraller system with 'n' units.

Paraller system with two units:

Two units are connected in parallel



x_1 → Successful operation

$\overline{x_1}$ → Un successful Operation

$P(x_1)$ → probability of sucess

$P(\overline{x_1})$ → probability of failure $(1-P(x_1))$
$(1-P(x_2))$

Both switches should fail. the probability of Unsuccessful operation of the system given By $(P(\overline{S}))$

$$P(\overline{S}) = 1 - P(S)$$

Case: 1 → Units are dependent on one another

$$P(\bar{S}) = P(\bar{x}) \cdot P(\bar{x_2})$$

$$= P(\bar{x_1}) \cdot P(\bar{x_2}/\bar{x_1})$$

Case: 2 (Units are independent of each other)

$$P(\bar{S}) = P(\bar{x_1} \text{ and } \bar{x_2})$$

$$= [1 - P(x_1)] \cdot [1 - P(x_2)]$$

$$P(\bar{S}) \Rightarrow 1 - P(S)$$

$$P(S) = 1 - P(\bar{S})$$

$$= 1 - [1 - P(x_1)] \cdot [1 - P(x_2)]$$

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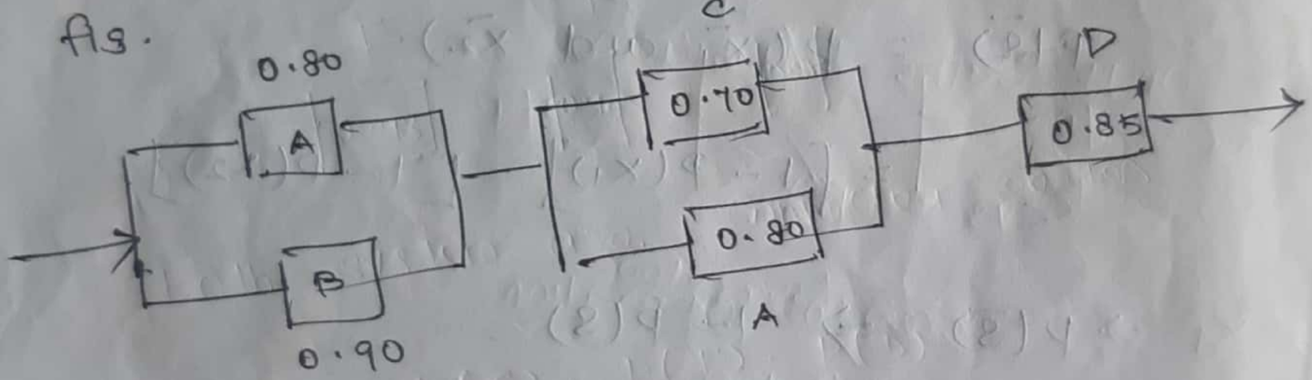
Mixed Configuration

* A mixed Configuration system is a system which several Elements are Connected in Series and parallel arrangements to perform a required system Operation.

problem:

Determine the reliability of the system whose Component reliabilities are shown in fig.



Soln:

A & B Connected in parallel.

A & B events denoted By E.

$$P(E) = P(A \text{ and } B)$$

$$= P(A) + P(B) - P(A \text{ and } B)$$

$$= 0.80 + 0.90 - (0.80 \times 0.90)$$

$$P(E) = \underline{\underline{0.98}}$$

C and A connected parallel denoted by F.

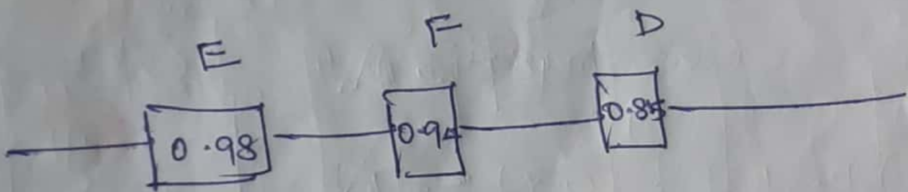
$$P(F) = P(C \text{ and } A)$$

$$\Rightarrow P(C) + P(A) - P(C \text{ and } A)$$

$$= 0.70 + 0.80 - (0.70 \times 0.80)$$

$$\boxed{P(F) = 0.94}$$

The system rearranged



Both system are connected series

$$P(S) = P(E) \cdot P(F) \cdot P(D)$$

$$= 0.98 \times 0.94 \times 0.85$$

$$\boxed{P(S) = 0.7830}$$

AND.

Problem MTTF

Problem: 2

Determine the MTTF for a mission time of 1000 hrs life if the test data on 10 such components gave times to fail as shown in table.

Component Number	1	2	3	4	5	6	7	8	9	10
time to failure in hours	807	820	810	875	900	837	850	790	866	815

Also find Reliability.

Given data:

$t = 1000$ hr

$N = 10$

To find:

→ MTTF

→ Reliability

Solution:

$$MTTF = \frac{1}{N} \sum_{i=1}^{n} t_i$$

$$\Rightarrow \frac{1}{10}\left[807+820+810+875+900+837+850+790+866+815\right]$$

$$MTTF \Rightarrow \underline{\underline{837}}$$

Reliability = $R(t) = e^{-\lambda x}$

$$\lambda = 1/MTTF$$

$$= \frac{1}{837} = \underline{\underline{0.0012}}$$

$$R(t) = e^{-(0.0012 \times 1000)}$$

$$R(t) = \underline{\underline{0.3012}}$$

Result:

MTTF = 837

R(t) = 0.3012

FAILURE TEST - DATA ANALYSIS

Important formula

1) failure Density (fd):

$$fd \Rightarrow \frac{\text{Num. of. failures during a given unit Interval}}{\text{Total Num of Items at the Beginning of the test.}}$$

$$fd = \frac{f}{IT \text{ (intial Item)}}.$$

2) failure Rate: (z)

$$z = \frac{\text{Num. of failure during a particular unit Interval}}{\text{Avg. popolation.}}$$

$$\Rightarrow \frac{① + ②}{2} \Rightarrow \text{Ans}.$$

$$= \frac{f}{\text{ans}} \Rightarrow \text{Result}.$$

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③ Reliability (R):

$$R = \frac{\text{Num of Survivors}}{\text{Total initial population}}$$

$$R = \frac{S}{NT}$$

Note: $R(t) = e^{-\lambda t}$

$\lambda \rightarrow$ failure Rate

$t \rightarrow$ Time period

④ Mean failure Rate (h):

$$h = \frac{(z_1 + z_2 + \dots z_n)}{T}$$

$z_1, z_2 \Rightarrow$ failure rate

$T \rightarrow$ Time

⑤ $MTTF = 1/\lambda$

$$MTTF = \frac{1}{N} \sum_{i=1}^{N} t_i$$

$$MTTF = \frac{1}{N} \sum_{k=1}^{l} k\, n_k \cdot \Delta t$$

$N \rightarrow$ No. of specimens

$t_i \rightarrow$ Time to failure for i^{th} specimen

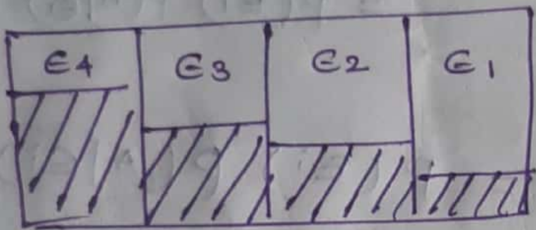
$\Delta t \rightarrow$ Time interval

$n_k \rightarrow$ No. of fail during k^{th}

Bayes theorem:-

$E_1, E_2, E_3, E_4 \Rightarrow$ Sensor.

unsatisfactory performance of the actuator of the Event - A.



∴ Based on figure we have formed.

$P(A) = P[A \text{ and } E_1]$ (or) $(A \& E_2)$ or $(A \& E_3)$ (or) $(A \& E_4)$

probability of the unsatisfactory performance

Since E_1, E_2, E_3, E_4 are mutually exclusive. Because all the Actuator uses a sensor supply (E_1, E_2, E_3, E_4).

$$\therefore P(A) = P(A \& E_1) + P(A \text{ and } E_2) + P(A \text{ and } E_3) + P(A \text{ and } E_4)$$

Using the multiplication rule,

We get,

$$P(A) = P(E_1)\,P(A|E_1) + P(E_2)\,P(A|E_2) + P(E_3)\,P(\tfrac{A}{E_3})$$

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$$+ P(E_4) \cdot P(A/E_4) \rightarrow ①$$

$$P(A) = \sum_{k=1}^{4} P(A/E_k) \cdot P(E_k) \rightarrow ②$$

Based on Observation

$$P(A \text{ and } E_i) = P(A)\, P(E_i|A)$$

Here

$E_k \rightarrow E_i$

$$= P(E_i)\, P(A|E_i)$$

$$\therefore P(E_i|A) = \frac{P(E_i)\, P(A|E_i)}{P(A)}$$

Here Substitue ①

We get,

$$P(E_i|A) = \frac{P(E_i)\, P(A|E_i)}{\sum_{k=1}^{4} P(E_k)\, P(A|E_k)} \rightarrow ③$$

∴ Important Bayes theorem.

For Example

$$P(E_1/A) = \frac{P(E_1)\, P(A/E_1)}{\sum_{k} P(E_k)\, P(A/E_k)}$$

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TIE SET AND CUT SET

Introduction:

* The methods of graph theory can be convienently used to analyze the Reliability aspects of a complex structure.

* The method of tie set and cut set are commonly used to complex Reliability.

* Assume input Node - I output Node-O

Tie-set:

* A tie set is a group of elements which forms a connection b/w input and output when traversed in the Arrow direction.

If no element is traversed more than once in tracing out tie-set. it is said to be a minimal tie-set

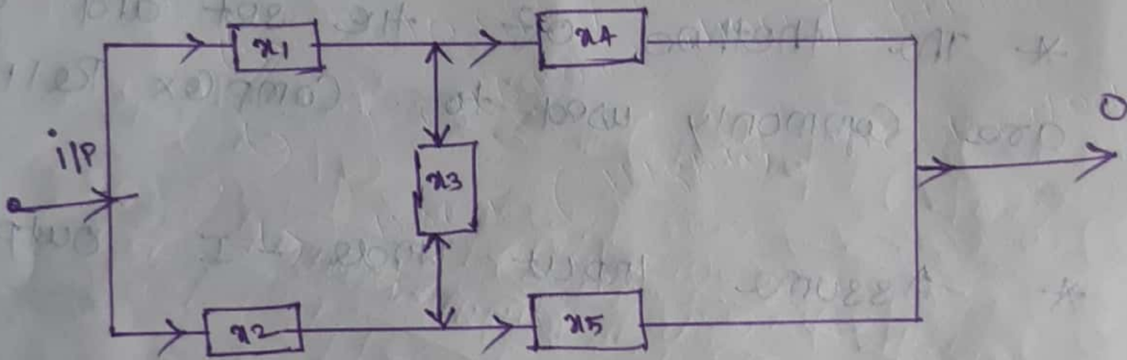
$i \rightarrow$ minimum tie-set

$T_1, T_2, T_3 \rightarrow$ Connector

$\therefore$ The system Reliability is given by,

$R(s) = P(T_1 \text{ (or) } T_2 \ldots T_i)$

$R(s) = P(T_1 + T_2 + \ldots T_i) \longrightarrow$ ①



$T_1 \Rightarrow x_1 x_4$

$T_2 \Rightarrow x_2 x_5$

$T_3 \Rightarrow x_1 x_3 x_5$

$T_4 \Rightarrow x_2 x_3 x_4$

cut set:

$C_1 \Rightarrow x_1 x_2$

$C_2 \Rightarrow x_4 x_5$

$C_3 \Rightarrow x_2 x_3 x_4$

$C_4 \Rightarrow x_1 x_3 x_5$

$C_5 \Rightarrow x_1 x_2 x_3$

$C_6 \Rightarrow x_3 x_4 x_5$

$\therefore R(s) = P(x_1 x_4 \text{ (or) } x_2 x_5 \text{ (or) } x_1 x_3 x_5 \text{ (or) } x_2 x_3 x_4)$

cut - set:

* A Group of Elements which if they are Removed from the block diagram will dis connect the input node from the Output node.

for above diagram:

$$R(s) = 1 - P(s)$$

$$= 1 - P(\bar{C_1} \text{ (or) } \bar{C_2}, \bar{C_3}, \bar{C_4})$$

$$R(s) = 1 - P(\bar{x_1}\bar{x_2} + \bar{x_4}\bar{x_5} + \bar{x_1}\bar{x_3}\bar{x_5} + \bar{x_2}\bar{x_3}\bar{x_4}).$$

Reliability Testing:

Objective of the Testing is to generate the failure data.

* Life testing may be for Engineering analysis, failure analysis, root Cause identification of failure mechanism. Wear limits, to Evaluate the life.

Type of Reliability test:

- Test with censoring / without censoring
- Test with / without Replacement
- Burn -in- test
- Life test
- storge / operation test
- Transportation / shipment test
- HALT (Highly Accelerated life testing)

Problem: 1

A test Conducted on 1000 Components used on a CNC m/c. the total duration of the test is 10 hours

- The Num of Components failing during Each hourly interval is noted and the result obtained are put in the followings tables.

Time (t)	No. of failures (f)	Cumulative failure (F)	No. of Survivors (S)	Failure Density (Fd)	Failure Rate (Z)	Reliability (R)
0		0	1000			1
	210			0.210	0.235	
1		210	790			0.790
	143			0.143	0.199	
2		353	647			0.647
	117			0.117	0.199	
3		470	530			0.530
	96			0.096	0.199	
4		566	434			0.434
	77			0.077	0.195	
5		643	357			0.357
	65			0.065	0.200	
6		708	292			0.292
	90			0.090	0.364	
7		798	202			0.202
	135			0.135	1.0037	
8		933	67			0.067
	50			0.050	1.190	
9		983	17			0.017
	17			0.017	2.000	
10		1000	0			0

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Explanation:

① Cumulative failure (F)

0 + 210 → 210

210 + 143 → 353

353 + 117 → 470

470 + 96 → 566

566 + 77 → 643

643 + 65 → 708

continued ...

② Num. of survivors (S)

S ⇒ Initial item − Cumulative failure

(IT) − (F)

(1000 − 0) → 1000

(1000 − 210) → 790

(1000 − 353) → 647

to be continued.

③ Failure density (fd)

$fd \Rightarrow \frac{\text{Num. of failure } (f)}{\text{Initial Item}} = \frac{210}{1000} = 0.210$

$\Rightarrow \frac{143}{1000} = 0.143$

$\frac{117}{1000} = 0.117$

$\frac{96}{1000} = 0.096$

to be continued.

④ Failure Rate (Z)

$Z = \left(\frac{1000 + 790}{2}\right) = 895$

$= f/895 = \frac{210}{895} = 0.235$

$\Rightarrow \left(\frac{790 + 647}{2}\right) = 718.5$

$Z = 143 / 718.5 = 0.199$

to be continued.

⑤ Reliability (R)

$R = \frac{S}{IT}$

$R = \frac{1000}{1000} = 1$

$= \frac{790}{1000} = 0.790$

$= \frac{647}{1000} = 0.647$

Hazard Plotting:

→ Constant Hazard

→ Linerly increasing

→ Weibull Model.

Constant Hazard:

$z(t)$ = Hazard rate

λ = failure rate

$$\boxed{z(t) = \lambda \times \text{constant}}$$

$$= \int_0^{\infty} z(t)\, dt$$

$$= \int_0^{t} \lambda\, dt$$

$$= \lambda t$$

$$R(t) = \exp\left[-\int_0^{\infty} z(t)\, dt\right]$$

$$= \text{Exp} \left[-\int_0^\infty \lambda \cdot dt \right]$$

$$= \text{Exp}(-\lambda t)$$

$$\boxed{R(t) = e^{-\lambda t}}$$

∴ WKT

$$F(t) + R(t) = 1$$

$$F(t) = 1 - R(t)$$

$$\boxed{F(t) = 1 - e^{-\lambda t}}$$

2) Linearly increasing

$$Z(t) = \lambda t \times \text{Constant}$$

$$= \int_0^\infty z(t)\ dt$$

$$= \int_0^t \lambda t \cdot dt$$

$$Z(t) = \frac{\lambda t^2}{2}$$

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$$R(t) = \mathrm{Exp}\left[- \int_0^{\infty} z(t)\, dt \right]$$

$$= \mathrm{Exp}\left[- \frac{\lambda t^2}{2} \right]$$

$$R(t) = e^{-\frac{\lambda t^2}{2}}$$

$$F(t) = 1 - R(t)$$

$$\boxed{F(t) = 1 - e^{-\frac{\lambda t^2}{2}}}$$

Weibull Model.

$$z(t) = \lambda t^m$$

Here

$$m > -1$$

$$Z(t) = \int_0^{\infty} z(t)\, dt$$

$$= \int_0^{t} \lambda t^m . dt$$

$$= \frac{\lambda t^{m+1}}{m+1}$$

$$R(t) = \text{Exp}\left[-\int_0^\infty z(t)\,dt\right]$$

$$= \text{Exp}\left[-\frac{\lambda t^{m+1}}{m+1}\right]$$

$$R(t) = e^{-\frac{\lambda t^{m+1}}{m+1}}$$

$$F(t) = 1 - R(t)$$

$$= 1 - \left[e^{-\frac{\lambda t^{m+1}}{m+1}}\right]$$

Ans

RELIABILITY. ① UNIT-II 1

FAILURE DATA ANALYSIS.

FAILURE DISTRIBUTION:-

- Reliability analysis – equipment / System – failure pattern.
- Some cases – not possible – Actual info of components.
- In that case failure distribution – found out
  ||
  Nature of the Failure.
- failure pattern – collected from Field data – history of equipment (or) System.
- In some case – parameter, equip/ System place .....
- that to be as hazard / failure
  used to calculate
  "failure distribution".
- It will be represented graphically, histogram, frequency (or) bar diagram.

Exponential distribution:-

For a continuous random Variable the probability density Function of exponential distribution can be expressed as

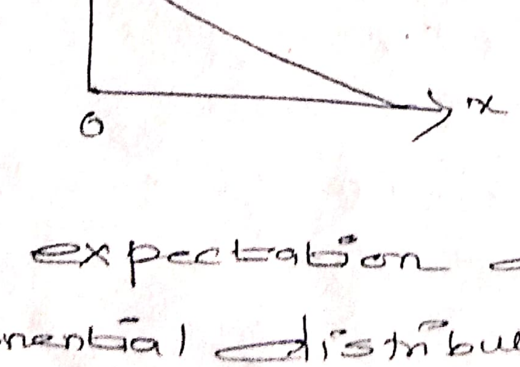
$$f(x) = \lambda e^{-\lambda x} \qquad x \geq 0, \lambda \geq 0.$$

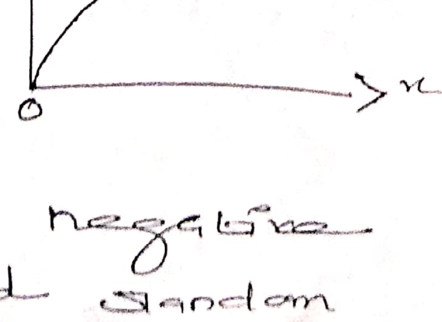
The cumulative distribution has the following expression

$$F(x) = 1 - e^{-\lambda x}$$

The complementary cumulative distri- bution function is expressed as

$$G(x) = e^{-\lambda x}$$





The expectation of negative exponential distributed random Variable is $E(x) = 1/\lambda$ which is

and the

Variance is $V(x) = 1/\lambda^2$ 2

Ex:-

The time between breakdown of m/c and life of an electric bulb etc....

Normal:-

* continuous probability Normal distribution.
* Countless - Application.
* Ideal represent - real life behaviour of many m/c components.
* real life behaviour expressed on mathematical frame......

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\left[(x-\mu)^2/2\sigma^2\right]}$$

$$(-\infty \le x \le \infty)$$

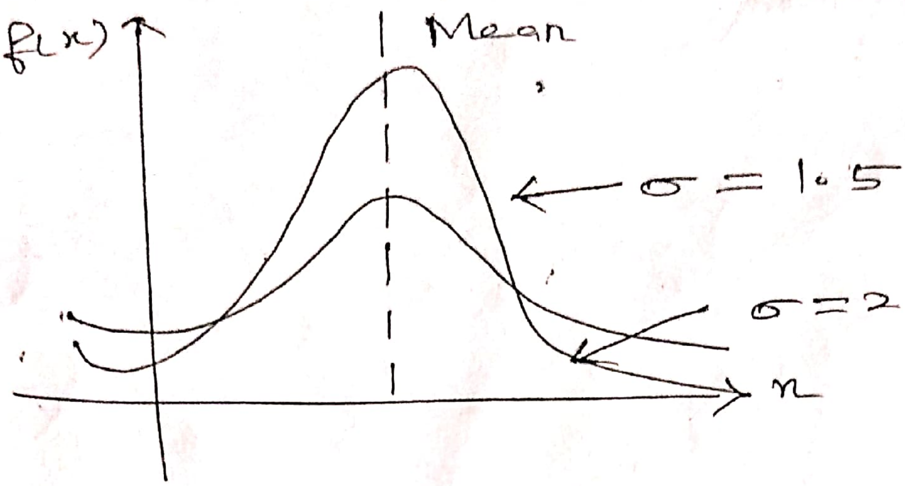
μ ⇒ unrestricted parameter → Specific Mean

σ ⇒ positive → Standard deviation

x ⇒ real number denoting random Variable.

The Cumulative density function

$$F(x) = \int_{-\infty}^{x} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left[\frac{y-\mu}{\sigma}\right]^2} dy.$$



It is apparent from above that f is a positive function, $e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)}$ being positive for any real number x. It shows that $\int_{-\infty}^{+\infty} f(x)\,dx = 1$. So, that $f(x)$ is a valid p.d.f.

The Standard normal random Variable $\Big\}$ $z = \dfrac{X-\mu}{\sigma}$

The density func. of the Standard normal random Variable is just

$$f(x) = \frac{1}{\sqrt{2\pi}}\, e^{-\left(\frac{x^2}{2}\right)} \quad -\infty \le x \le$$

$E(x) = \mu$

$Var(x) = \sigma^2$

3) **Gamma Distribution :-**

* reliability analysis - electrical components.
* Complex engineering system.
* . The shape factor of a number of failures takes places at a constant rate, then the failures of a system as a whole is distributed according to gamma distribution.

$$f(x) = \frac{\lambda^n x^{n-1} e^{-\lambda x}}{\Gamma(n)} \quad \text{for}$$

$E(x) = k/\lambda$
$V(x) = k/\lambda$

$x, \lambda, n \geq 0.$

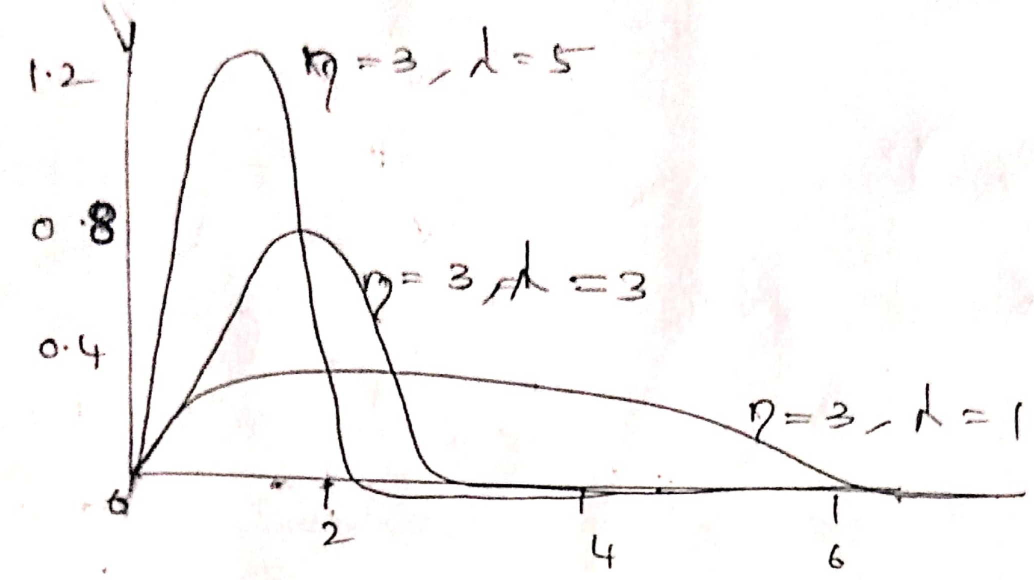
where,

λ = Scale factor.
n = Shape factor.

Gamma function $\Gamma(x) = \int_0^\infty x^{n-1} e^{-x} dx$

Show the gamma density function for different value of λ and $n=3$

When $n=1$ the density function reduce to $f(x) = \lambda e^{-\lambda x}$



The expectation & variance of gamma distributed random variable x is expressed as

$$E(x) = \frac{\eta}{\lambda}$$

$$V(x) = \frac{\eta}{2\lambda}$$

Weibull Distribution:-

- two parameters.
- three parameters.

$$f(x) = \frac{m}{\theta}\left(\frac{x}{\theta}\right)^{m-1} e^{\left[-\left(\frac{x}{\theta}\right)^{m}\right]}$$

$x \geq x_0$

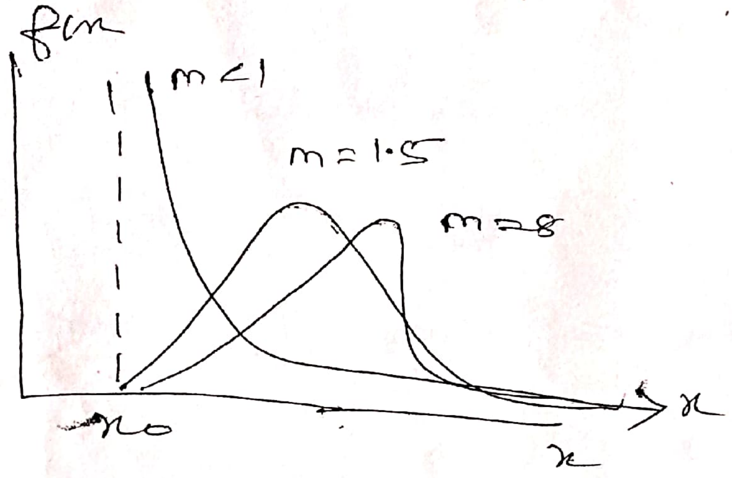
$$f(x) = \frac{m}{\theta - x_0}\left(\frac{x - x_0}{\theta - x_0}\right)^{m-1} e^{\left[-\left(\frac{x - x_0}{\theta - x_0}\right)\right]}$$

$x \geq$

(F) where ϕ = characteristic (or) scale value $(\phi \geq 0)$. 4

m = shape parameter.

x_0 = guaranteed value of x $(x_0 \geq 0)$



The cumulative distribution fun

$$F(x) = e^{\left[\frac{(x-x_0)^m}{(\phi - x_0)}\right]}.$$

But putting $x_0=0$, the cumulative distribution 'E' is obtained for two parameters and expression will be

$$F(x) = 1 - e^{\left[-\left(\frac{x}{\phi}\right)^m\right]}.$$

The mean of Weibull distributed random variable is

$$\mu_x = x_0 + (\phi - x_0)\,\Gamma\left(1+\frac{1}{m}\right)$$

& standard deviation is expressed as

$$\sigma_x = (\theta - x_0)\left[\Gamma\left(1+2/m\right) - \Gamma^2\left(1+ \right.\right.$$

Γ is a gamma function.

The median of the distribution is expressed as

$$X_{med} = x_0 + (\theta - x_0)(\ln 2)^{1/m}$$

mode is represented by

$$X_{mode} = x_0 + (\theta - x_0)\left(\frac{m-1}{m}\right)^{1/m}.$$

(*) Applicable to find life time & waiting time.

probability:

(*) Reliability $\rightarrow$ associated with probability

(*). It measures the uncertainty about the Occurance of a particular event (or) a set of events and is expressed between zero and

(4) probability plot:-

⇒ It is graphical techniques for accessing whether (or) not a data set a follow a given distribution such as normal (or) Weibull.

⇒ The data are plotted againt a theoretical distribution - such - point should form approximately a straight line.

⇒ It can be Generated for several competing distribution to see which provides the best fit.

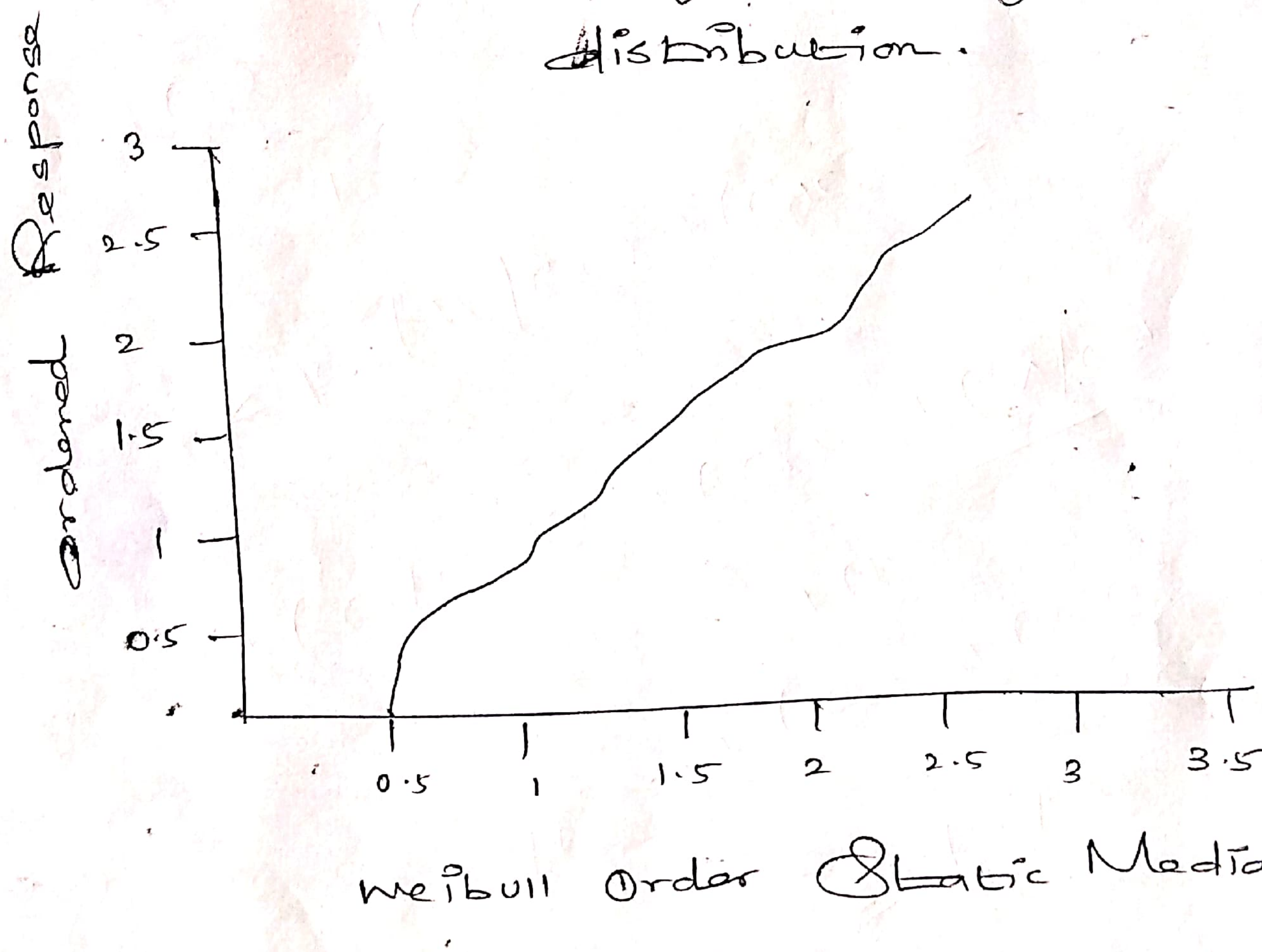
⇒ PPCC (probability plot correla - tion co-efficient) plot provides an excellent method for estimating the shape parameter.

This data is a set of 500 Weibull random number with a shape parame - ter = 2, location parameter = 0, and scale parameter = 1. The

Weibull probability plot includes that the Weibull distribution does in fact fit these data well.

The probability plot is formed by

* Vertical axis:- Ordered response values.
* Horizontal axis:- Order Static Medians for the given distribution.



The Order Static Medians

$$N_i = G(U_i)$$

$U_i \Rightarrow$ Uniform Order Static Medians.

$G \Rightarrow$ percent point function.

The percent point function is the inverse of the cumulative distribution function. (probability that x is less than (or) equal to some value).

The Uniform Statistic median are

$$M_i = 1 - M_n \text{ for } i = 1.$$

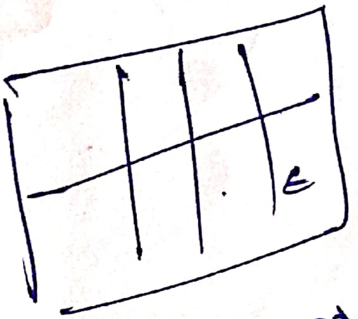
$$M_i = \frac{(i - 0.3175)}{(n + 0.365)} \text{ for } i = 1, 2, 3 \ldots n-1$$

$$M_i = 0.5^{(1/n)} \text{ for } i = n.$$

Hazard plotts:

* Similar to probability plotts.
* visually examining distributional model assumption for reliability data.
* It consist of a plot of the cumulative hazard $H(t_i)$ versus the time t_i of the i-failure

Bayes theorem



$P(A) = P(A$ and

$P(A) = P(E_1)$

E

By

$\sum_{k=1}^{n} P(A|E_k)\, P(E_k) \rightarrow$

$$P(E_i|A) = \frac{P(E_i)\, P(A|E_i)}{P(A)}$$

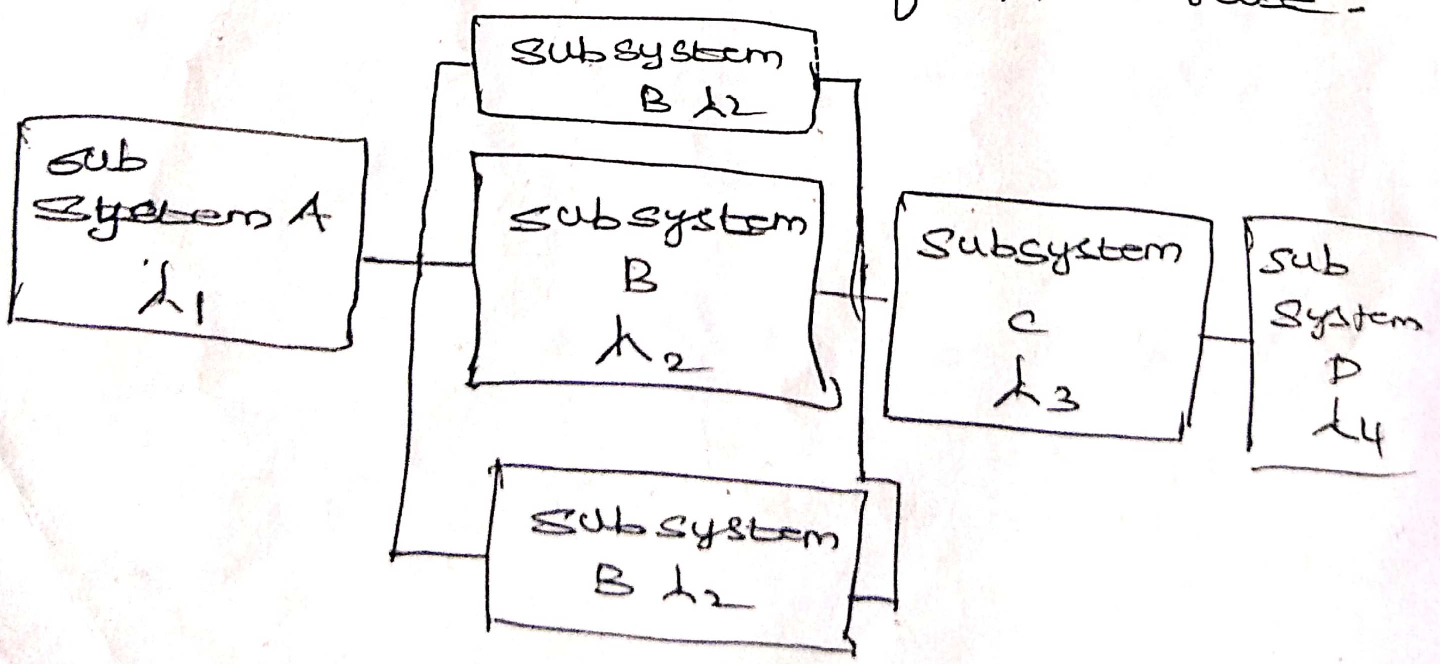
$$P(E_i|A) = \frac{P(E_i)\, P(A|E_i}{\sum_{k} (P(E}$$

UNIT-III

RELIABILITY PRODUCTION MODELS

RBD Approach:-

- RBD ⇒ Reliability block Diagram.
- It is diagrammatic method for showing how components reliability contributes to the success (or) failure of a complex system.
- It also known as Dependence Diagram (DD).
- It drawn as a series of blocks connected in parallel (or) series configuration.
- Each component represent component of the system with a failure rate.

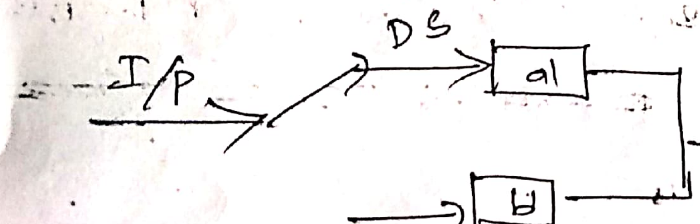


(*) RBD ⇌ success tree ⇌ fault tree

(By Appling morgan's theorem)

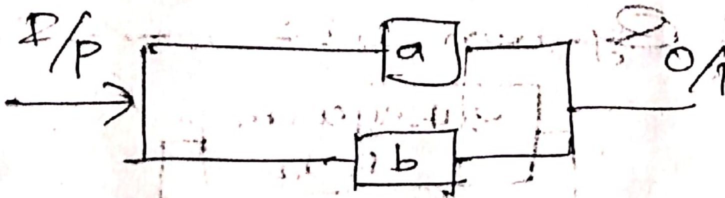
Stand by Redundancy :- (*)

In the parallel system, all the channels (or) paths are active from the beginning of the operation of the system till its failure. In a stand by system, all the paths are not active at the same time.



→ Decision switch

(DS switch connect at a1)



$$p(s) = 1 - p(\bar{a} \ \& \ \bar{b})$$

$$p(s) = 1 - p(\bar{a}) \times p(\bar{b} \mid \bar{a})$$

↳ probability failure of element b

$$p(s) = 1 - (1-p)(1-p)$$

$$\Rightarrow 2p - p^2$$

$$p(s) = 1 - p(\bar{a}' \ \& \ \bar{b}')$$

RELIABILITY UNIT-III

RELIABILITY PREDICTION MODELS.

Series Systems:

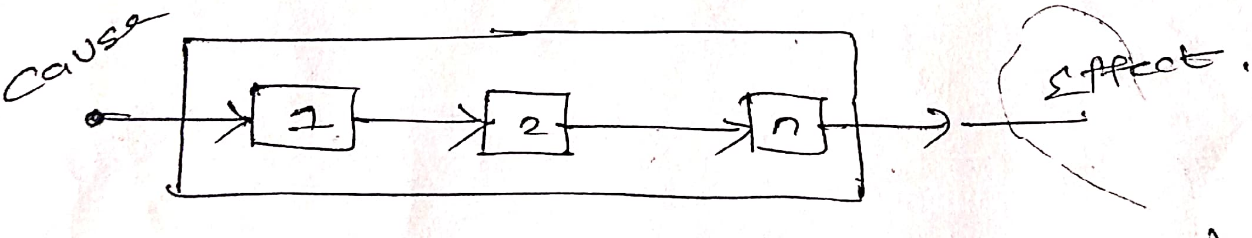
The Simplest combination of unit that forms a System is a Series combination.

The System consist of n Unit which are connected in Series. which are represented as

$x_1, x_2, \ldots\ldots x_n$.

Probabilities by

$p(x_1), p(x_2), \ldots\ldots p(x_n)$.



$$p_1(S) = p(x_1 \text{ and } x_2 \text{ and } \ldots\ldots \text{and } x_n).$$

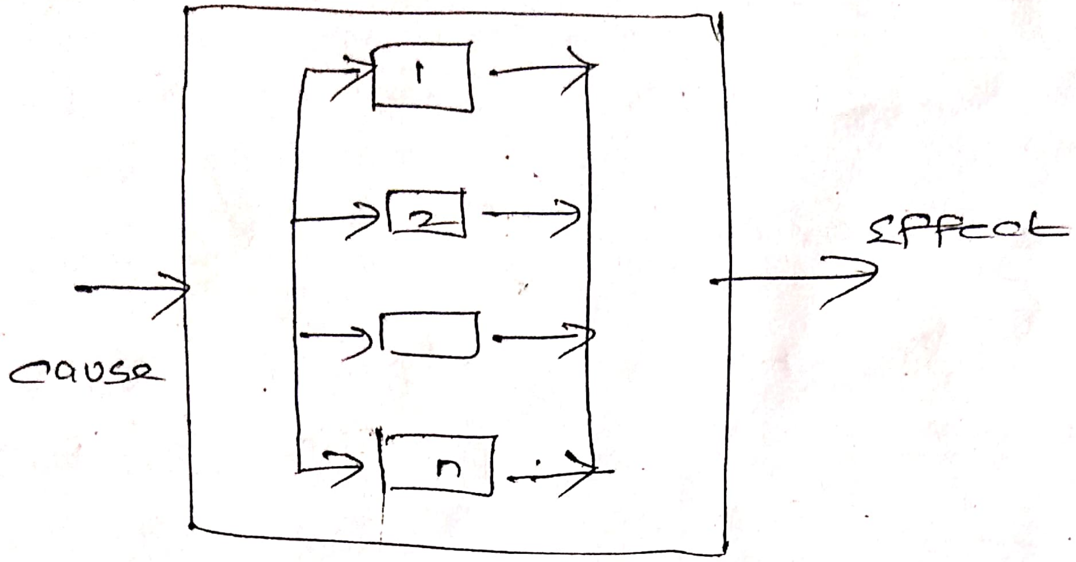
$$= p(x_1) \times p(x_2|x_1) \times p(x_3|x_1 \text{ and } x_2) \times \ldots\ldots$$

$$p(x_n|x_1 \text{ and } x_2 \text{ and } \ldots\ldots x_{n-1}).$$

$$p(S) = p(x_1) \times p(x_2) \times \ldots\ldots p(x_n).$$

Parallel System:

The Successful Systems are



$$p(\bar{S}) = p(\bar{x}_1 \text{ and } \bar{x}_2 \text{ and } \ldots \text{ and } \bar{x}_n)$$

$$= p(\bar{x}_1) \times p(\bar{x}_2 | \bar{x}_1) \times p(\bar{x}_3 | \bar{x}_1 \text{ and } \bar{x}_2) \times \ldots$$

$$\times\, p(\bar{x}_n | \bar{x}_1 \text{ and } \bar{x}_2 \text{ and } \ldots \text{ and } \bar{x}_{n-1})$$

$$p(\bar{s}) = p(\bar{x}_1) \times p(\bar{x}_2) \times \ldots\, p(\bar{x}_n) =$$

$$= 1 - p(s)$$

$$p(s) = [1 - [1-p(x_1)] \times [1-p(x_2)] \times \ldots$$

$$\times [1-p(x_n)].$$

$$p(s) = 1 - p(\bar{s})$$

$$= 1 - [1-p(x)]^n.$$

$$= 1 - p(\bar{a}') \times p(\bar{b}' \mid \bar{a}')$$

$$p(s) = p(1 - \log_e p).$$

Markov Analysis:-

$$\text{up} \xrightarrow{\lambda} \text{down}$$

$$P_0(t + \Delta t) = (1 - \lambda \Delta t) \cdot p_0(t) + 1 \cdot 0 \, p_1(t).$$

$$p_1(t + \Delta t) = 1 \cdot p_1(t) + \lambda \Delta t \cdot p_0(t)$$

State transition Matrix

	0	1
0	$1 - \lambda \Delta t$	$\lambda \Delta t$
1	0	1

$$\frac{P_0(t + \Delta t) - p_0(t)}{\Delta t} = -\lambda p_0 t$$

$$\frac{P_1(t + \Delta t) - p_1(t)}{\Delta t} = \lambda p_1 t$$

$$\left.\frac{P_0(t + \Delta t) - P_0(t)}{\Delta t}\right|_{\Delta t \to 0} = P_0'(t)$$

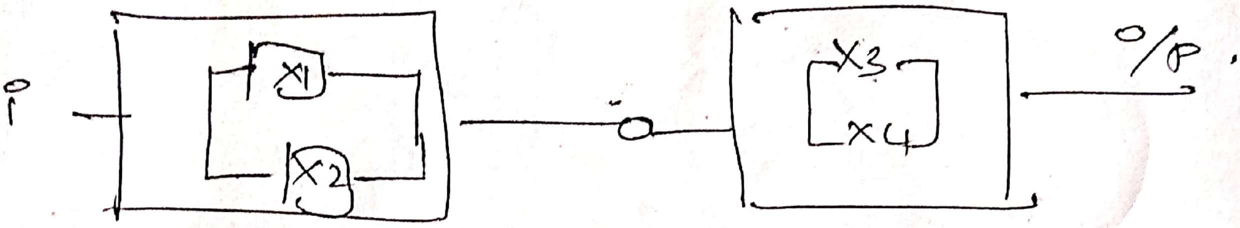
$$\left.\frac{P_1(t + \Delta t) - p_1(t)}{\Delta t}\right|_{\Delta t \to 0} = P_0'(t)$$

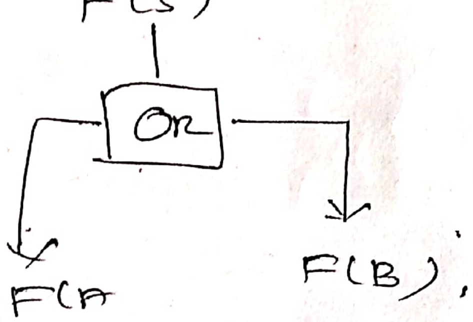
$$P_0(t) = e^{-\lambda t}$$

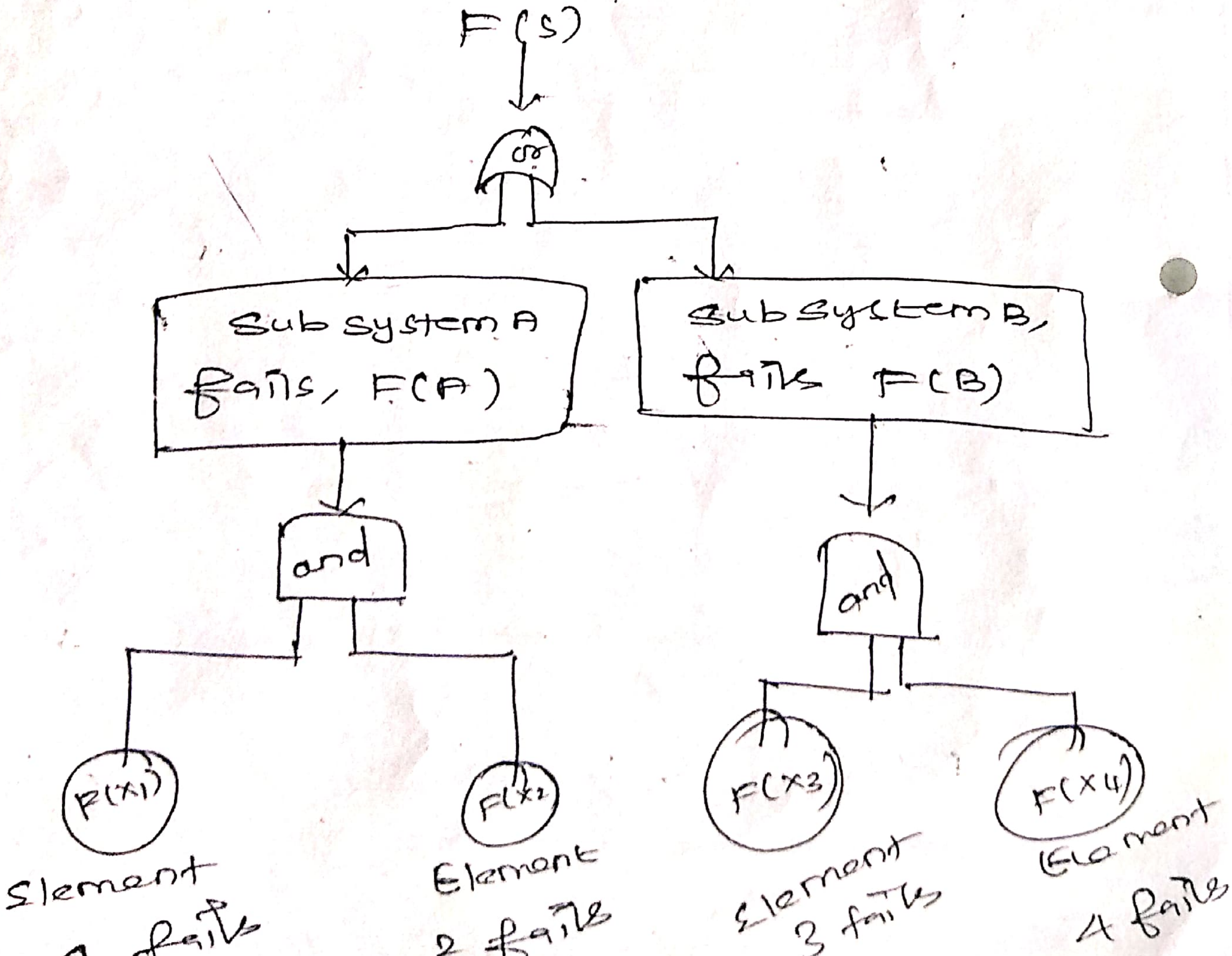
Fault tree Analysis:-

Two sub system connected in Series

A & B.

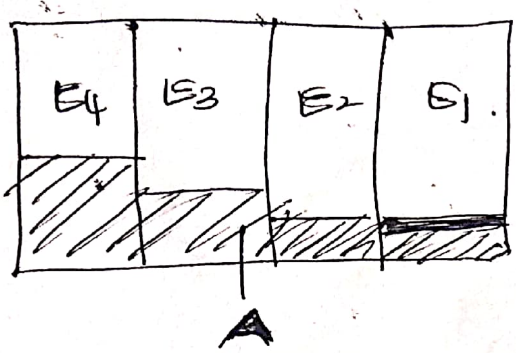






Bayes' Theorem:-

A



Prob

$p(A) = p(A \,\&\, E_1) + p(A \,\&\, E_2) + p(A \,\&\, E_3) + p(A \,\&\, E_4)$ → ②

$$p(A) = p[(A \text{ and } E_1) \text{ or } (A \text{ and } E_2) \text{ or } (A \text{ and } E_3) \text{ or } (A \text{ and } E_4)] \rightarrow ①$$

P

$E_1, E_2, E_3, E_4 \Rightarrow$ Mutually exclusive

$$p(A) = p(E_1)p(A/E_1) + p(E_2)p(A/E_2) + p(E_3)p(A/E_3) + p(E_4)p(A/E_4)$$

By Multiplication rule,

$$= \sum_{k=1}^{4} p(A/E_k)\, p(E_k) \rightarrow ③$$

Earlier causes:-

$$p(A \text{ and } E_i) = p(A)p(E_i/A)$$
$$= p(E_i)\, p(A/E_i).$$

Therefore,

$$P(E_i/A) = \frac{p(E_i)\, p(A/E_i)}{p(A)} \rightarrow ④$$

Sub ③ & ④.

... what is the reliability of the control system.

$$p(s) = 0.98 \times 0.92 \times 0.9$$

$$= 0.811.$$

(3). If the System consists of n identical Units ~~of~~ in Series, and if each Unit has a reliability factor P, determine the System reliability factor, assuming that all Units function ~~inden~~ independently. Approximately this when the reliability factor, assuming that all Units function Idependently Approximately this when the reliability factor is high.

If q is the probability of failure of each Unit, then $p = 1-q$ and the System reliability is

$$p(s) = p \times p \times \cdots \times p = p^n = (1-q)^n.$$

If q is very small, this express

$$\sum_{k=1}^{4} p(E_k)\, p(A/E_k).$$

This is the important ①.

$$p(E_1|A) = \frac{p(E_1)\, p(A/E_1)}{\sum_{k} p(E_k)\, p(A/E_k)}$$

problems:- (unit - III).

1. An electronic equipment is operated by four dry cells, each giving 1.5 volts. The cells are connected in series. The probability of the successful operation of each cell under the given operating condition is 0.90. Calculate the reliability of the power system.

Events are independent.

$p(s) = 0.9 \times 0.9 \times 0.9 \times 0.9 = 0.656.$

2. In an hydraulic cntrl system, the connecting linkage has a reliability factor of 0.98 and the value which has to operate with in a certain time limit has a reliability factor of 0.92. The pressure sensor which actuates the linkage has a reliability factor of 0.90. Assume that all the three elements namely the actuator, the linkage and the hydraulic valve, are

(4). A System has ten identical components connected in series. It is desired that the system reliability be 0.95. Determine how good each component should be.

$$p(s) = 0.95 = p^{10}.$$

So, $p^{10} = 0.95$

Taking log on both sides,

$$\log 0.95 = 10 \log p.$$

$$= 0.9949.$$

(5). A parallel ~~plate~~ system is composed of ten identical independent components. If the system reliability $p(s)$ is to be 0.95, how poor can be the component be?

$$p(s) \Rightarrow 1 - [1 - p(x)]^{10}$$

$$0.95 = 1 - [1 - p(x)]^{10}.$$

$$[1 - p(x)]^{10} = 1 - 0.95 = 0.05.$$

$$1 - p(x) = 0.7419$$

$$p(x) = 0.2581$$

p(s) = System reliability.

reliability factor is high } = 0.95.

(5) The power of supply to the operati-ng unit of a hospital is provided by a generator whose failure state follows an exponential distri-bution law with parameter $\lambda_1 = 0.005$ per hour. A standby battery unit is coupled through a decisi switch which has a reliability rd = 0.90 calculate the reliability of the power supply system for a mission time of 10 hours if the battery failure rate follows a distribution law with parameter $\lambda_2 = 0.001$ per hour.

$$R(10hr) = \exp(-\lambda_1 t) + rd \frac{\lambda_1}{\lambda_2 - \lambda_1} \left[\exp(-\lambda_1 t) - \exp(-\lambda_2 t)\right]$$

$$= e^{-0.05} + 0.9 \times \left(\frac{0.005}{0.001 - 0.005}\right)$$

$$(e^{-0.05} - e^{-0.01})$$

Bayes.

A Manufacturer of an electronic actuator buys his requirement of sensor from four different suppliers. on an average firm E_1 supplies 30% of his requirements, firm E_2 supplies 25% of the requirements firm E_3 supplies 22% and firm E_4 supplies 23% of the requirements. A Quality test carried out on the sensors supplied by each firm reveals that.

6%	of	the	supplies	from	E_1	are	below std
8%	"	"	"		E_2	"	"
10%	"	"	"		E_3	"	"
12%	"	"	"		E_4	"	"

$p(E_1)\,p(A/E_1) = (0.3)(0.06) = 0.018.$

$p(E_2)\,p(A/E_2) = (0.25)(0.08) = 0.020.$

$p(E_3)\,p(A/E_3) = (0.22)(0.10) = 0.022.$

$p(E_4)\,p(A/E_4) = (0.23)(0.12) = 0.0276.$

$$p(E_1/A) = \frac{0.018}{(0.018 + 0.020 + 0.022 + 0.0276)}$$

$$= \frac{0.018}{0.0876} = 0.205.$$

(2). A Manufacturing concern specializing in high-pressure relief values subjects every value to particular acceptance test before certify-ing it as fit for use. Over a period of time, it is observed that 95% of all values measure passed the test. How ever, the acceptance test adopted is found to be only 98% reliable consequently, a value certified as fit for use has a probility of 0.02 being faulty. what is the probability that a satisfactory value will be pass the test? (5)

$p(E/A) = 0.98$, $p(\bar{E}/\bar{A}) = 0.98$

$$p(E/\bar{A}) = 1 - 0.98 = 0.02$$

$$p(A) = 0.95.$$

$$p(E) = p(E/A)\,p(A) + p(E/\bar{A})\,p(\bar{A}).$$

$$= 0.98 \times 0.95 + 0.02 \times 0.05$$

$$= 0.932.$$

$$p(A/E) = \frac{p(E/A) \cdot p(A)}{p(E)}$$

$$= (0.98 \times 0.95) / 0.932$$

$$= 0.931 / 0.932$$

$$= 0.999.$$

3). A construction company fabricates a large number of steel frameworks to support driving mechanism for dredging m/c's. Each frame work is designed to carry a load of 2KN. The design calculations have, however assumed a load of 2.5KN for the driving mechanism. But due to variations in the Quality of the material used and the fabrication process, it is estimated that ~~that~~ ~~for~~ the reliability of the framework to carry 2.5KN is only 0.80. In Order to arrive at more realistic value for the reliability of the framework to carry 2.5KN a series of tests are conducted. The ~~tests~~ reveal that 50% of the structures that fails to carry 2.5KN fails at loads less than 2.3KN. The problem is to utilize this info to arrive at a reliability value for